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TECHNIQUES FOR REMOTE SENSING OF MISSILE POSITION AND ATTITUDE

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This report presents a mathematical analysis of a three station laser tracker and a two station laser tracker concept for measuring the position and attitude of spinning rockets and missiles. An assessment is included of the errors of attitude measurements which result from the geometrical location of the rocket relative to the laser tracking stations.

The two station system includes a concept developed by Ballistic Research Laboratories for the yawsonde attitude measurement system. Whereas the yaw- over

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sonde utilizes the sun as the energy source, this system will utilize two laser trackers. Two arrays of roof reflectors which are skewed to the rocket axis will be employed on the rocket to reflect the laser beam. A mathematical analysis is presented for this system. The determination of the attitude angles requires the solution of two simultaneous differential equations. Since multiple solutions to these equations can be obtained, additional analysis of this system is warranted.

CONTENTS

1. Introduction

Measurement of vehicle dynamics during flight on test ranges has been approached using both ground-based and onboard instrumentation. Onboard platforms and accelerometers require telemetry systems or recorders resulting in relatively expensive instrumentation which must be considered expendable in most tests. Onboard solar aspect sensors have been developed for spinning missiles which can be used to infer yawing, pitching, and rolling motion of flight vehicles. The most successful of these is the yawsonde developed at the Ballistic Research Laboratories[1,2]. Many flight systems are tested using only ground-based instrumentation employing primarily photographic techniques. Data reduction in these cases is essentially all manual with attendant high cost and relatively long data reduction time.

Two approaches for the determination of position and attitude of flight vehicles are described in this report. These systems involve ground-based laser transmitter/detector stations and expendable retro-reflecting elements located on the vehicle. When compared to current ground-based photographic techniques for determining both position and attitude, the proposed systems offer several advantages including: lower recurring costs; improved accuracy; automated data reduction and applicability in low ambient light situations, and in situations where the trajectory of the vehicle is not well established prior to flight.

This report describes the two concepts and presents some limited results that were obtained from system analysis studies.

2. The Three Station System

a. System Concept

This system includes three ground based transmitter/detector tracking stations – each incorporating one pulsed and one continuous wave (CW) laser of different frequencies. Onboard the vehicle, two different types of retroreflecting arrays are required.

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[1]Mermagen, W. H. and Clary, W. H., The Design of a Second Generation Yawsonde, Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland, BRL Memorandum Report No. 2368, 1974 (Unclassified).

[2]Mermagen, W. H., "Measurements of the Dynamical Behavior of Projectiles over Long Flight Paths," J. Spacecraft and Rockets, Vol. 8, No. 4, April 1971.

One type of array is composed of either conventional corner cubes, reflective tapes and/or paints which have the property of retroreflecting a portion of the collimated incident beam back parallel to itself regardless of the orientation of the reflective surface. This array forms a retroreflecting band located on the perimeter of the vehicle body at one axial position. Illumination of and reflection by this retroreflecting band will then form a conventional laser radar tracking system[3,4].

A series of roof type prisms as shown in Figure 1 form a second array which is also mounted on the vehicle's surface. This particular reflector array will be designated here as the single plane corner reflector. A plane, which passes through the center of all the reflecting surfaces of the roof type array and is also normal to all these surfaces, will be called the retroreflection plane. Collimated light incident on the single plane corner reflector and contained in its retroreflection plane ($\beta = 90°$) is reflected back to the source. The single plane corner reflector is mounted on the surface of the vehicle in such a manner that its retroreflection plane contains the roll axis of the vehicle. An illustration of an instrumented vehicle is provided in Figure 2.

As the vehicle flies downrange, it is tracked with the laser radar which positions a CW laser to provide a continuous illumination of the vehicle and also provides the information for determining the vehicle position as a function of time. During each revolution of a spinning vehicle, a CW laser pulse is returned to every tracking station. The time interval between the pulses returned to three separate tracking stations provides enough data for determining the vehicle attitude. A mathematical description of this system is presented in the next section.

It should be noted here that the missile position can be determined using triangulation from two or more tracking stations. Range determination with the use of a pulse laser is therefore not required for a multistation system. A single CW laser could be used for both attitude and position determination if discrimination between the two types of reflections were possible.

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[3] Pell, Kynric M. and Conrad, Robert G., Preliminary System Study of Roll Rate Determination Utilizing a Corner Reflector, US Army Missile Command, Redstone Arsenal, Alabama, July 1971, Report No. RD-TM-71-8 (Unclassified).

[4] Conard, Robert G., Pell, Kynric M., and Nydahl, John E., Measurement of Missile Position and Attitude by Lasers, US Army Missile Command, Redstone Arsenal, Alabama, July 1975, Report No. RD-TR-76-7 (Unclassified).

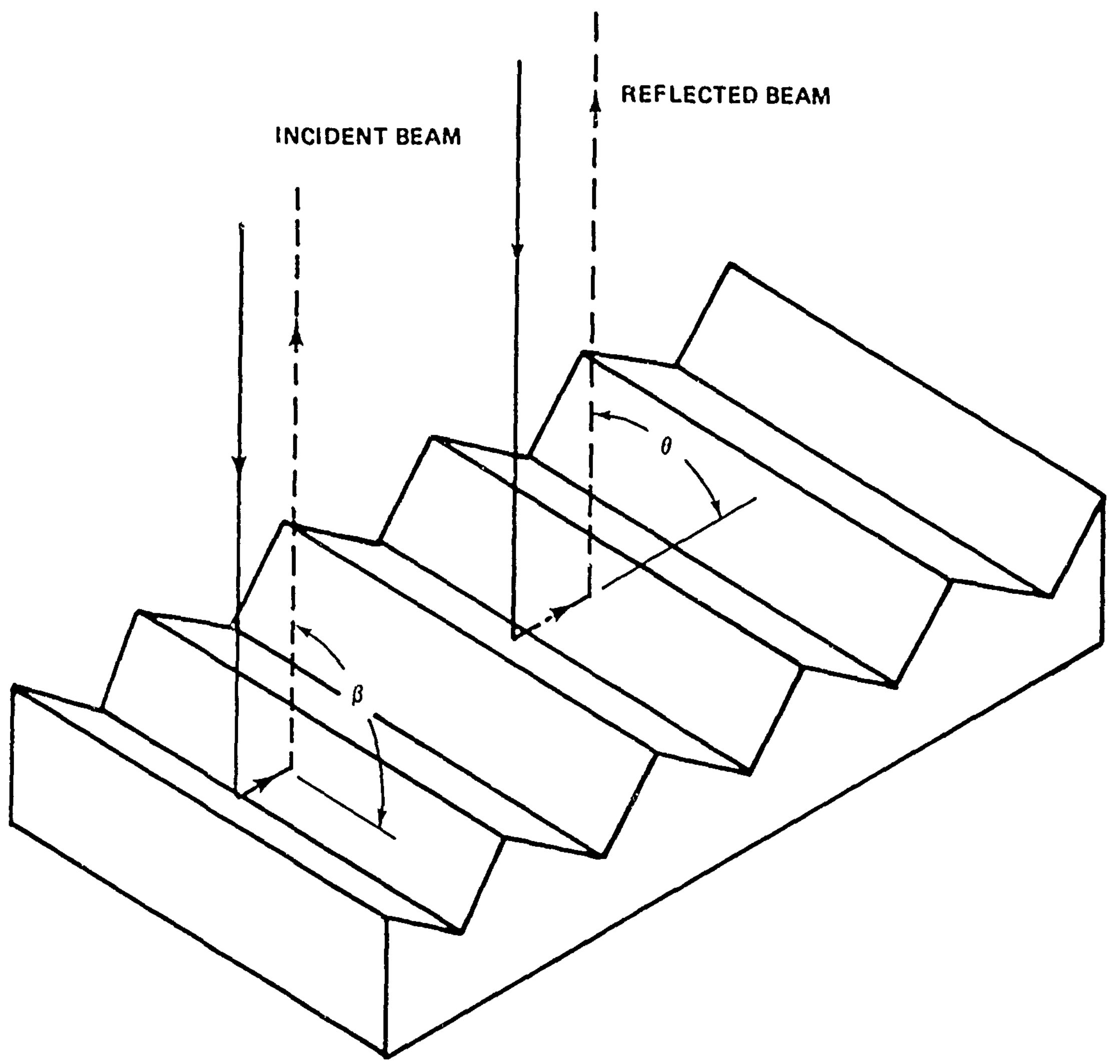

Figure 1. Single plane corner reflector (roof prism).

b. Mathematical Description of the System

This analysis assumes constant values for the vehicle position, pitch, yaw, and roll rate during the time it takes for a CW pulse to be received at each of the three tracking stations. The impact of these assumptions will be treated as an error in the subsequent error analysis. A vehicle which is fixed in space, rotating at a frequency ω, equipped with a previously described single plane corner reflector and illuminated with a CW laser will retroreflect the incident CW beam to a detector co-located with the CW laser source and thereby generate a periodic signal of frequency ω.

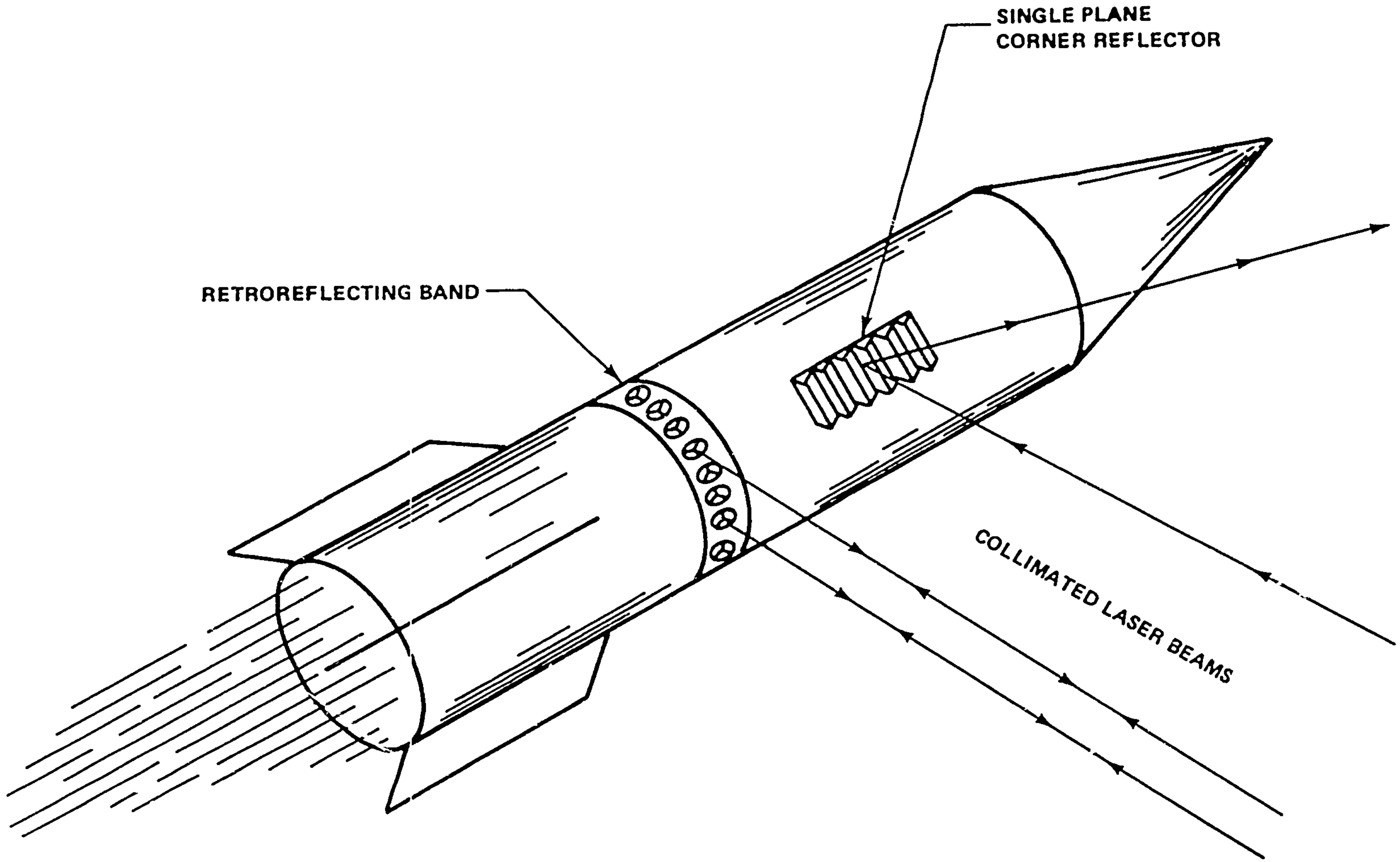

Figure 2. Illustration of vehicle showing retroreflectors.

The mathematical description may be facilitated by defining three coordinate systems. An earth fixed Cartesian system (Figure 3) is defined with the following characteristics:

1) The origin is located at the launch site.

2) y positive is in the downrange direction.

3) x is crossrange.

4) z is positive in the vertical upward direction.

5) (x,y,z) forms a right hand system.

A vehicle based coordinate system (Figures 3 and 4) is defined such that:

1) The origin is located at the vehicle center of gravity.

2) coincides with vehicle roll axis with the positive direction toward the nose.

3) η is perpendicular to ω and parallel to the x-y plane of the earth-fixed system.

4) ξ is perpendicular to η and ω such that (η,ω,ξ) is a right-hand system.

An intermediate system (x', y', z') which is just a direct translation of the earth fixed system to the vehicle center of gravity is also defined as shown in Figure 4.

One approach to analysis of the system involves a description of the locations of the ground transmitter/detector stations in terms of the vehicle based (η,ω,ξ) coordinates. Let $\bar{R}_i$ be the position vector of the $i^{\underline{th}}$ ground tracking station in the earth fixed system, $\bar{R}_m$ be the position vector of the vehicle in the earth fixed system, and $\bar{R}_i'$ be the position vector of the $i^{\underline{th}}$ ground tracking system in the x', y', and z' system. That is,

$$\bar{R}_i' = \bar{R}_i - \bar{R}_m \quad . \tag{1}$$

The position vector of the $i^{\underline{th}}$ ground tracking station in the vehicle based coordinate system $(\eta_i, \omega_i, \xi_i)$ will be denoted as $\bar{\psi}_i$. The relationship between $\bar{\psi}_i$ and $\bar{R}_i'$ is given by

$$\bar{\psi}_i = \{A\} \bar{R}_i \tag{2}$$

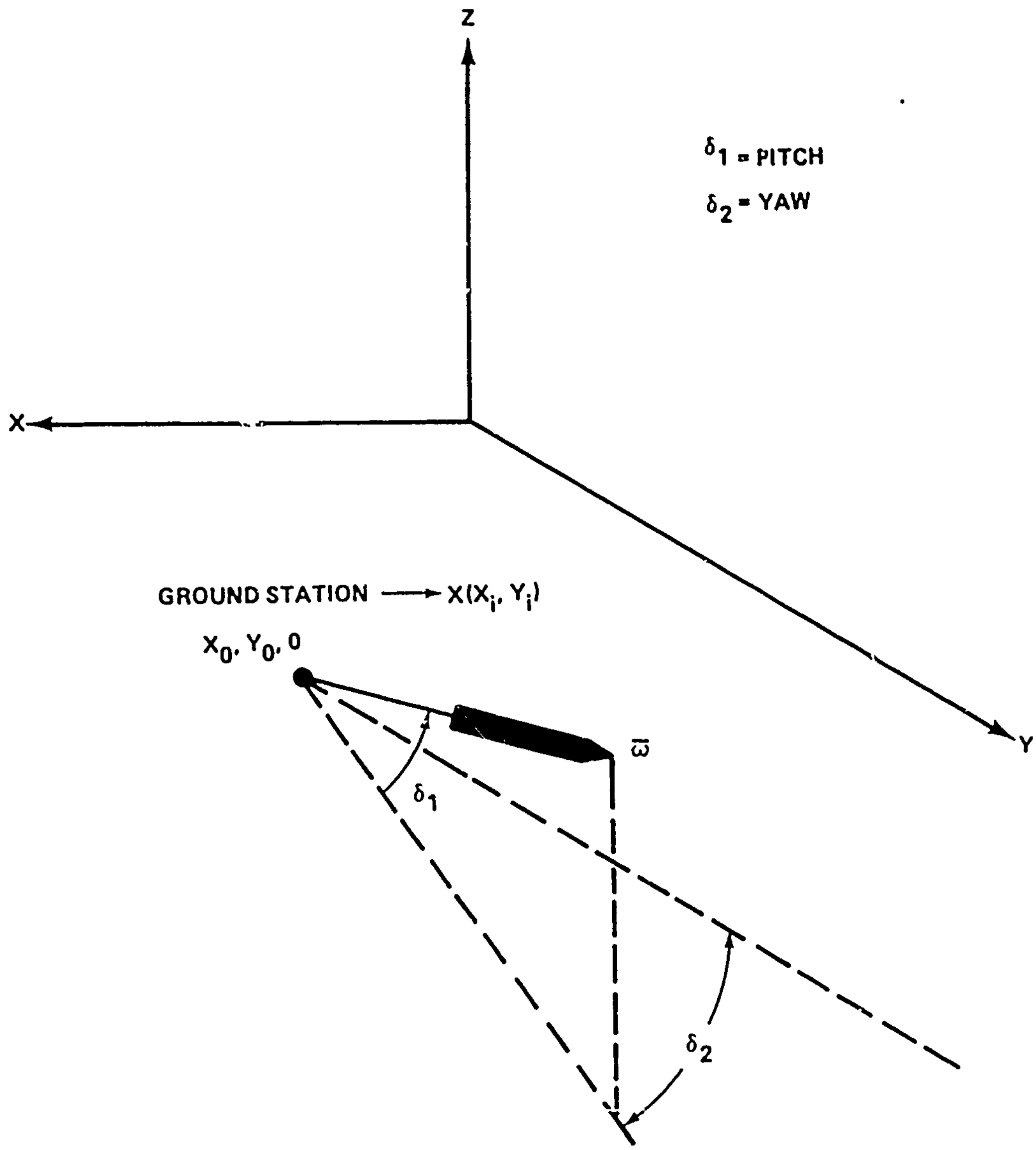

Figure 3. X,Y,Z coordinate system.

where $\{A\}$ is the coordinate transformation matrix,

$$\begin{bmatrix} \cos\delta_2 & -\sin\delta_2 & 0 \\ (\sin\delta_2\cos\delta_1) & (\cos\delta_2\cos\delta_1) & \sin\delta_1 \\ (-\sin\delta_2\sin\delta_1) & (-\cos\delta_2\sin\delta_1) & \cos\delta_1 \end{bmatrix} , \qquad (3)$$

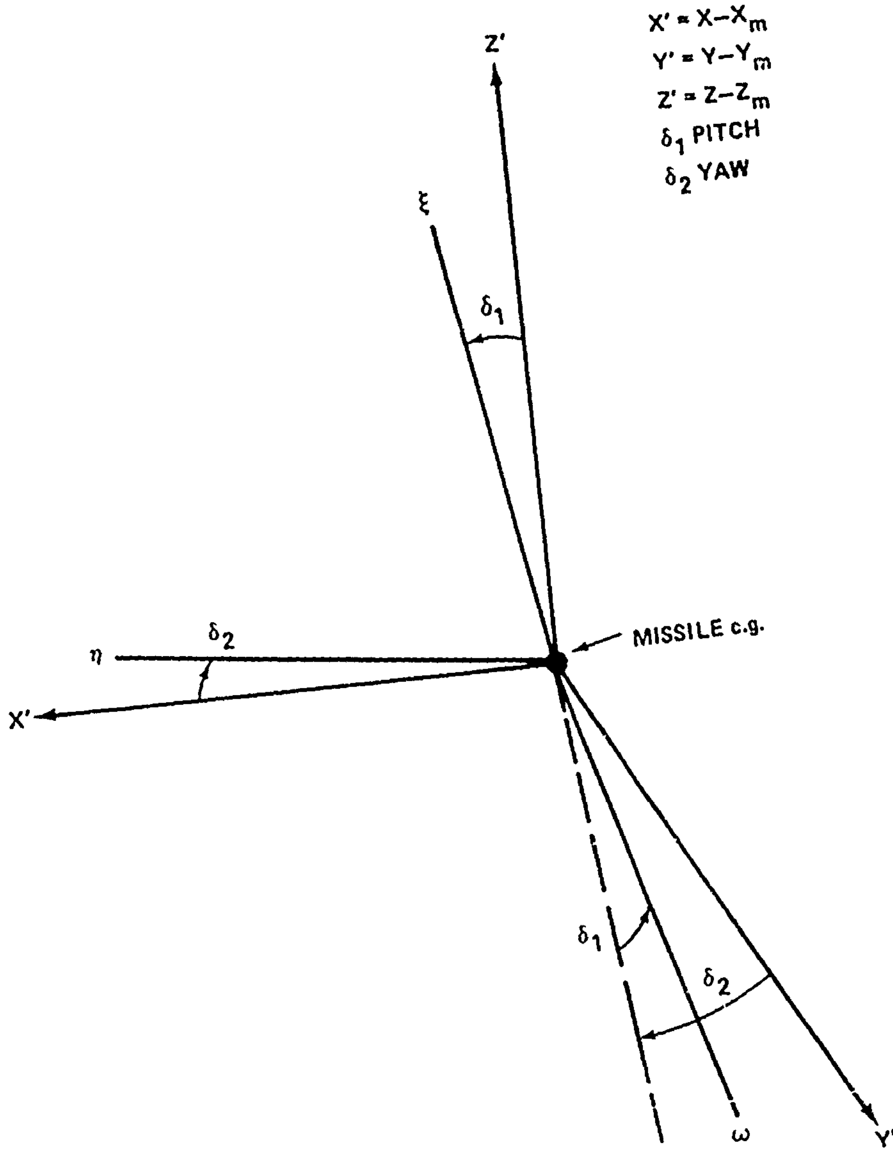

Figure 4. X', Y', Z' coordinate system.

and δ_1 is the missile pitch and δ_2 is the yaw relative to earth fixed coordinate system (Figure 5). When the coordinate transformation (2) is performed, one obtains the equations

$$\eta_i = \cos(\delta_2)\ (x_i - x_m) - \sin(\delta_2)\ (y_i - y_m) \tag{4}$$

$$\omega_i = \sin(\delta_2)\ \cos(\delta_1)\ (x_i - x_m) + \cos(\delta_2)\ \cos(\delta_1)\ (y_i - y_m) \tag{5}$$

$$+ \sin(-_1)\ (z_i - z_m)$$

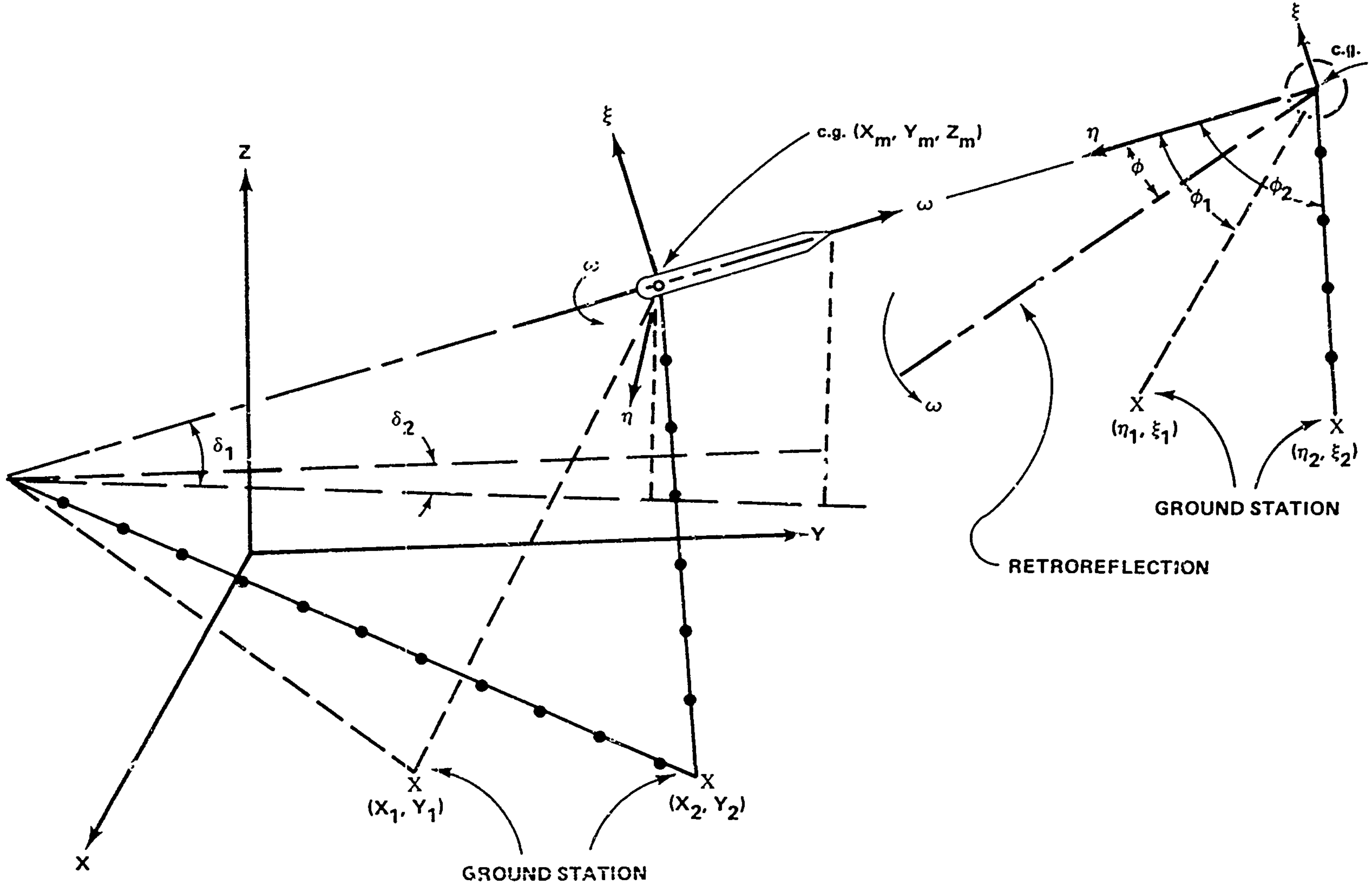

Figure 5. Geometric analysis.

$$\xi_i = -\sin(\delta_2)\sin(\delta_1)(x_i - x_m) - \cos(\delta_2)\sin(\delta_1)(y_i - y_m) + \cos(\delta_1)(z_i - z_m) \tag{6}$$

which gives a relation between the vehicle's measured position ($x_i - x_m$, $y_i - y_m$, $z_i - z_m$) and the 5 unknown parameters: pitch (δ_1), yaw (δ_2), and the station coordinates (ξ_i, ω_i, η_i).

The two additional relationships needed to close this system of equations can be developed by determining the angle that the retroreflection plane makes with the η coordinate which is denoted as Φ in Figure 5. Since the roll axis ω is contained in the retroreflection plane, this angle can be expressed in terms of the vehicle based coordinates η and ξ. Further, the angle swept out by the retroreflection plane as it moves between two ground stations(i and j) can be expressed in terms of these same two coordinates as indicated in Figure 5.

$$\Phi_i - \Phi_j = \tan^{-1}\left(\frac{\xi_i}{\eta_i}\right) - \tan^{-1}\left(\frac{\xi_j}{\eta_j}\right) \tag{7}$$

This can be equated to the product of the roll rate (ω) and the measured time lapse (Δt_{ij}) between the pulses received at two of the ground stations when the retroreflection plane passes through them.

$$\Phi_i - \Phi_j = \omega \Delta t_{ij} \tag{8}$$

Any two of the three possible time differences which can be formed in this manner (Δt_{12}, Δt_{13} and Δt_{23}) could be used to close the above set of equations (4, 5, and 6). Choosing Δt_{12} and Δt_{13} and making the appropriate substitutions reduces this set of equations to the following two equations:

$$\Delta t_{12} = \frac{1}{\omega}\left\{\arctan\left[\frac{-\sin(\delta_2)\sin(\delta_1)(x_2 - x_m) - \cos(\delta_2)\sin(\delta_1)(y_2 - y_m) + \cos(\delta_1)(z_2 - z_m)}{\cos(\delta_2)(x_2 - x_m) - \sin(\delta_2)(y_2 - y_m)}\right] - \arctan\left[\frac{-\sin(\delta_2)\sin(\delta_1)(x_1 - x_m) - \cos(\delta_2)\sin(\delta_1)(y_1 - y_m) + \cos(\delta_1)(z_1 - z_m)}{\cos(\delta_2)(x_1 - x_m) - \sin(\delta_2)(y_1 - y_m)}\right]\right\} \tag{9}$$

and

$$\Delta t_{13} = \frac{1}{\omega}\left\{\arctan\left[\frac{-\sin(\delta_2)\sin(\delta_1)(x_3 - x_m) - \cos(\delta_2)\sin(\delta_1)(y_3 - y_m) + \cos(\delta_1)(z_3 - z_m)}{\cos(\delta_2)(x_3 - x_m) - \sin(\delta_2)(y_3 - y_m)}\right] - \arctan\left[\frac{-\sin(\delta_2)\sin(\delta_1)(x_1 - x_m) - \cos(\delta_2)\sin(\delta_1)(y_1 - y_m) + \cos(\delta_1)(z_1 - z_m)}{\cos(\delta_2)(x_1 - x_m) - \sin(\delta_2)(y_1 - y_m)}\right]\right\}. \tag{10}$$

ω can be determined from the time interval between successive pulses received at a ground station and the same pulse information is used to calculate the time differences Δt_{12} and Δt_{13}. The vehicle's position relative to the ground stations $(x_i - x_m, y_i - y_m, z_i - z_m)$ is measured by the tracking part of the system. The pitch (δ_1) and yaw (δ_2) of the vehicle can therefore be determined by the simultaneous solution of the previously noted equations.

These two equations are transcendental and may possibly have multisolutions but this does not seem to be the case in the investigations carried out to date.

c. Ground Station Position Optimization

Three major considerations in the optimization of ground station locations for the laser systems were used:

1) In view of the high accelerations and resultant large velocities associated with rockets and missiles, the stations must be located so that the required angular tracking rate of the azimuth/elevation mount will be within the mount's capabilities.

2) The efficiency of a single plane corner reflector (roof prism) decreases rapidly as the angle of incidence (θ in Figure 1) diverges from 90°. Thus, the intensity of the reflected beam decreases from approximately 100% of the incident beam at normal incidence (θ = 90°) to zero at θ = 45°. This dictates ground station locations which provide working angles relative to the missile axis (if the reflecting surfaces are arranged symmetrically with respect to the missile axis) ranging from 60° to 120° in order to have the reflected beam intensity greater than approximately 50% of the incident beam.

3) The third constraint arises from the fact that the equation for Δt_{ij} in terms of pitch and yaw cannot be solved explicitly for pitch and yaw. Simultaneous numerical solution of the two equations is facilitated if the ground stations are arranged so that each of the partial derivatives $\partial\Delta t_{ij}/\partial\delta_1$ and $\partial\Delta t_{ij}/\partial\delta_2$ is either positive or negative over the entire dynamic range of the vehicle flight. These pitch and yaw sensitivity expressions are developed in the next session.

To check the static model developed above in a dynamic situation, a six degree of freedom computer simulation of a particular rocket (ARROW) was modified to include subroutines that provide return pulse data and sensitivity values for a simulated laser attitude system. After a number of simulations it was determined that a static missile position could be used to determine the sign of the sensitivities developed from the Δt_{ij} equations. This simplification results in a considerable reduction in computer time since the trajectory simulation does not have to be exercised for every ground station geometry, and further, it allows a more systematic study of the interaction of missile position and ground station geometry with respect to the various sensitivities.

A computer program was developed which produced the various sensitivities for fixed missile position and attitude and a particular ground station geometry. The missile position was varied over the grid:

$x_m = 0.$

$y_m = 10, 100, 200, 300, 400, 500, 1000, 1500, 2000$ (ft).

$z_m = y_m$ 10 (ft).

The attitude was varied over the grid:

$\delta_1 = 0$ to 10 in increments of 1°.

$\delta_2 = -5$ to 5° in increments of 1°.

Roll rate was held constant at 3020°/sec. Ground station positions were varied over the domain:

$x_i = -3000$ to 3000 ft in increments of 1000 ft.

$y_i = 0$ to 3000 ft in increments of 1000 ft.

$z_i = 0.$

Pitch and yaw sensitivities were calculated for 121 pairs of pitch and yaw values for each combination of ground station locations.

For the yaw equation (i.e., the equation in which pitch is held constant in an iterative solution), the program printed out the arithmetic mean, the maximum, and the minimum values for pitch and yaw sensitivities if the yaw sensitivities were all either greater than or less than zero. If values of both signs occurred, the next combination of tracking stations was checked without providing a computer printout of the case which failed criteria 3. Printout was also suppressed if the missile pointed directly at a ground station due to a singularity in the sensitivity equations for this case. For the pitch equation, a similar output was generated.

This study indicated that a wide range of ground station positions were available for attitude determination but no single combination of three ground station positions was found which would provide coverage over the complete dynamic range of the missile as specified. In retrospect, a negative pitch attitude during the first 2000 feet of flight appeared unrealistic, and since it was determined that these values were excluding a large number of possible station locations, the investigation was repeated constraining δ_1 range from 0 to 10° in increments of 1°.

The results from the previous investigation indicated that the location (3000, 0, 0) would be a reasonable position for the first ground station. Using this location for station one, 86 combinations for the other two ground station positions providing positive results in the sense of criteria 3 were developed. An examination of the mean of the calculated yaw sensitivities indicated that maximum values occurred when one ground station was located along the line y = 3000 feet. Figure 6 presents a plot of yaw sensitivity as a function of a ground station position along the line y = 3000 where the other two stations are located at (3000, 0, 0) and (3000, 2000, 0). In addition, plots of the pitch and yaw sensitivity envelopes as a function of the downrange missile position for two other combinations of tracking station positions are shown in Figures 7 and 8.

Considering the second constraint in the light of the preceding results, it is apparent that at least two retroreflecting prisms with different orientations of the single plane corner reflectors will be required on a vehicle in order to provide coverage downrange for 2000 feet. Although actual constraints on angular tracking rate (criteria 1) were not investigated, the angular rate for a third station located anywhere along the line y = 3000 would be less than that required of stations 1 and 2 which apparently can be handled with off-the-shelf equipment.

d. <u>Error Analysis</u>

In a dynamic situation where a retroreflector equipped rocket flies a trajectory such that the laser pulses are returned to a number of ground stations, the time interval between reception of pulses at any two ground stations is influenced by changes in the missile position and attitude between the times of reception of the pulses by the two stations, as well as changes in the roll rate during the time interval.

In the analysis that was presented in Section 2.b. for determining the missile pitch and yaw from the points in time when pulses were reflected back to one of the three gound stations, it was assumed the missile position, attitude and roll rate did not change during the time intervals associated with the reception of pulses at the three ground stations. In the simplified analysis to follow, the observed time interval Δt_{ij} between any two station pulse receptions is treated

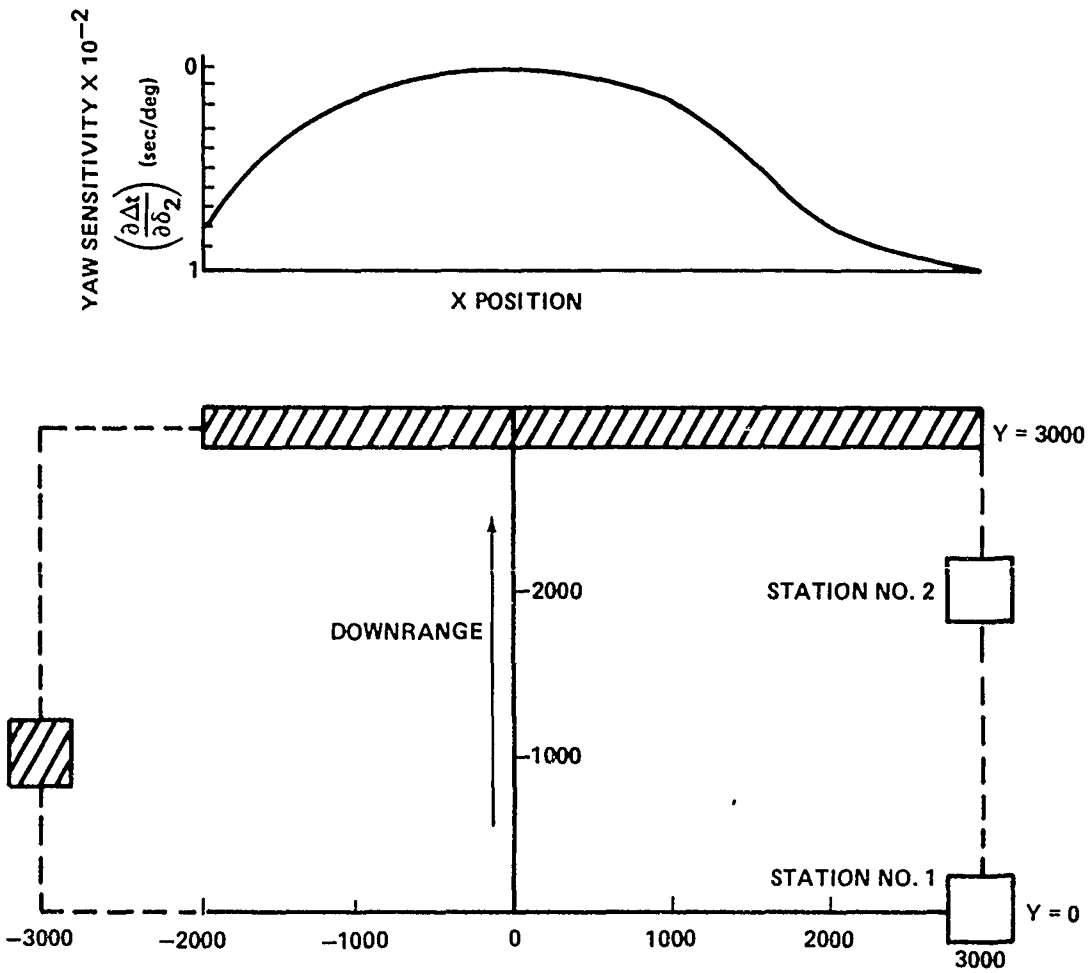

Figure 6. possible third station locations based on criteria 3.

as a nominal time interval $\overline{\Delta t}_{ij}$ plus a number of perturbation terms. $\overline{\Delta t}_{ij}$ is defined as the time which would be observed if the missile were fixed in space at a position midway between the positions associated with the missile at the times of reception for the two pulses; possessed a pitch and yaw equal to the mean of the values associated with the vehicle at the times of reception for the two pulses; and had a roll rate equal to the mean of the values associated with the vehicle at the times of reception for the two pulses. The perturbation terms are developed assuming that the influencing parameters (variables) are independent and further, that the influence can be represented as a linear variation around the nominal value; in other words, as the first term in the Taylor Series expansion. Mathematically this may be represented as

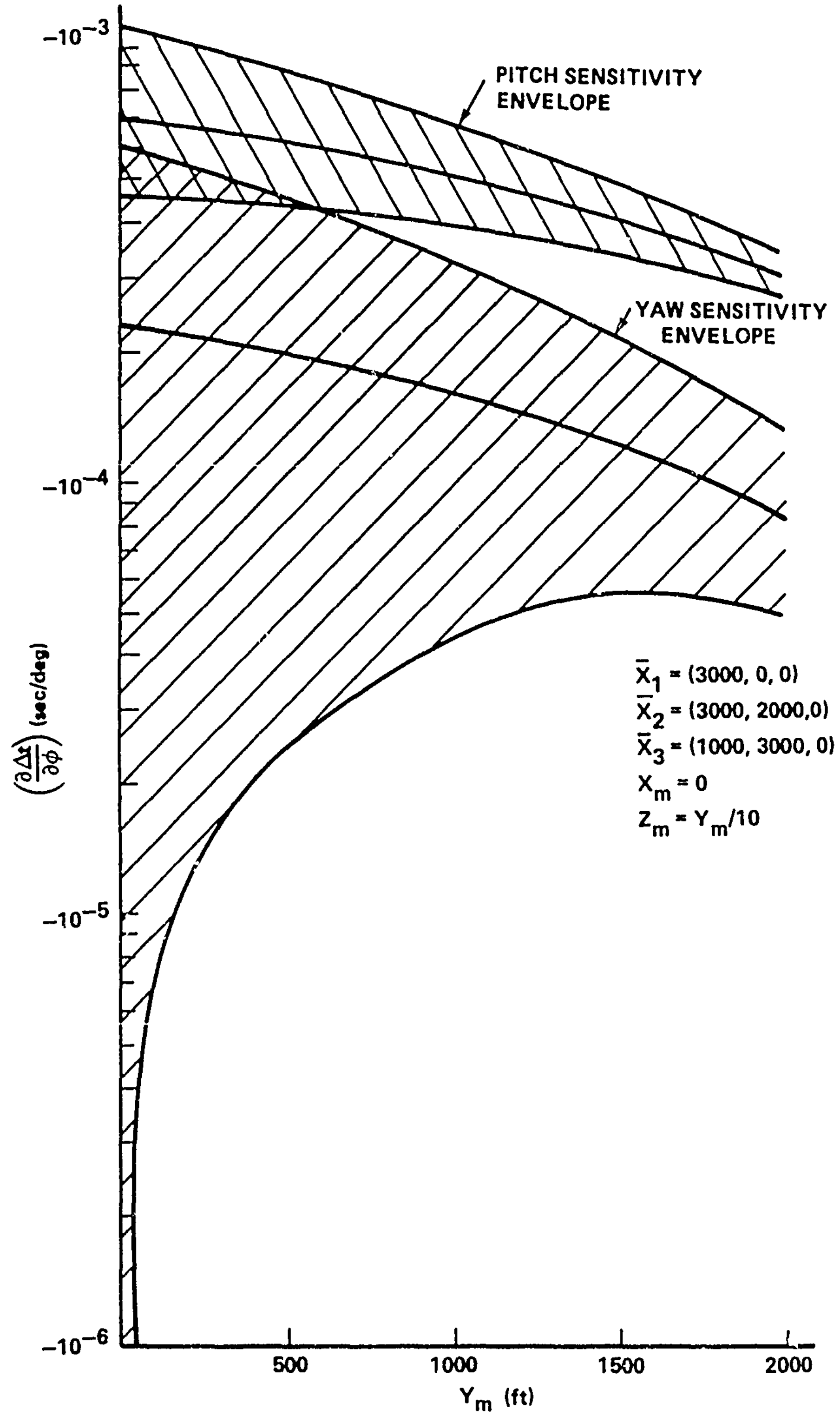

Figure 7. Sensitivity envelopes for X_3 = (1000, 3000, 0).

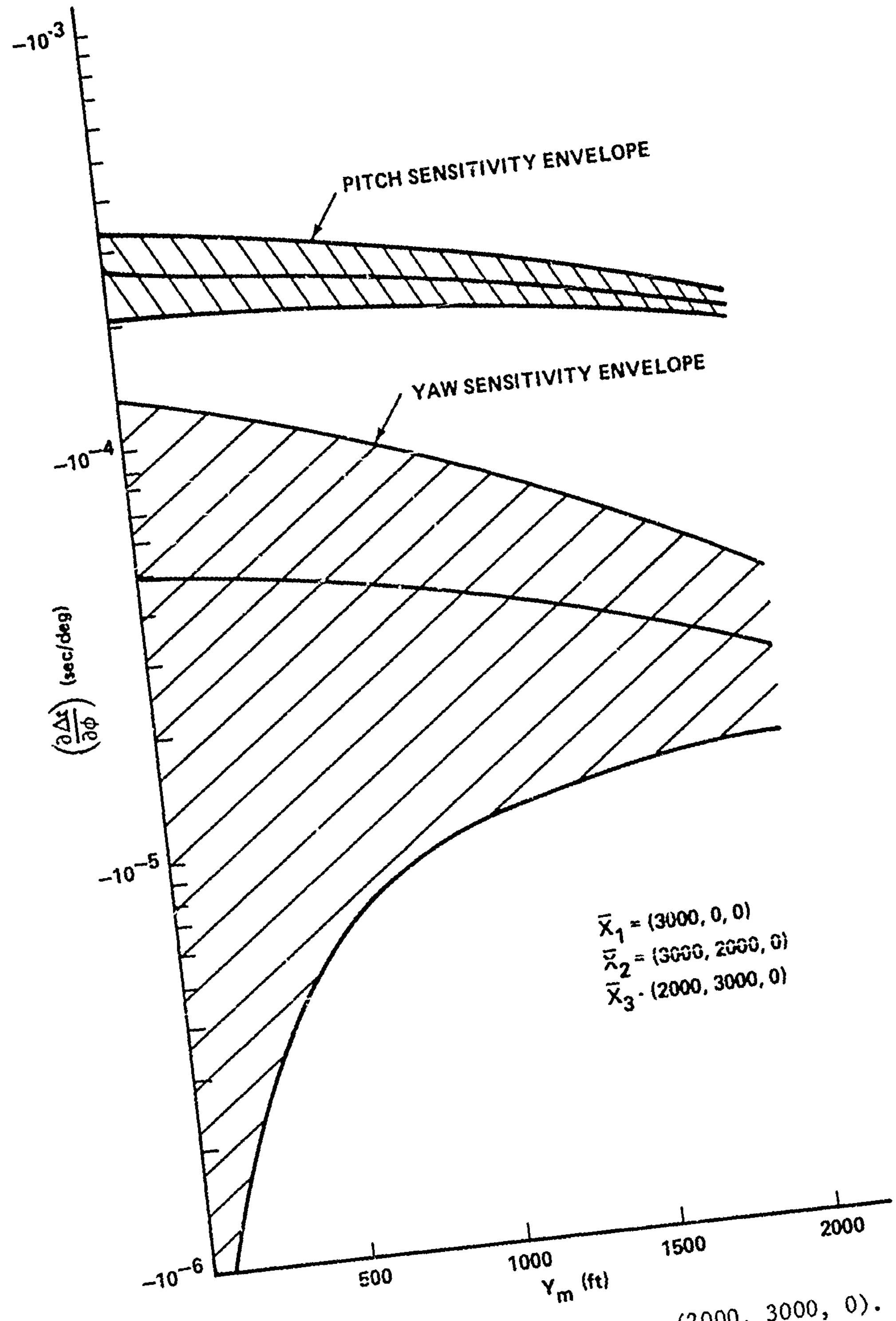

Figure 8. Sensitivity envelopes for X_3 = (2000, 3000, 0).

$$\Delta t_{ij} = \overline{\Delta t}_{ij} + \left\{ \sum_{k=1}^{7} \left[\left(\frac{\partial \Delta t_{ij}}{\partial \phi_k} \right)_{t=\bar{t}} (\bar{\phi}_k - \phi_k) \right]^2 \right\}^{1/2} \tag{11}$$

where $\frac{\partial \Delta t_{ij}}{\partial \phi_k}$ is the sensitivity of the time interval Δt_{ij} to variations of parameters ϕ_k which includes: $\phi_1 = \delta_1$, $\phi_2 = \delta_2$, $\phi_3 = \omega$, $\phi_4 = x_m$, $\phi_5 = y_m$, $\phi_6 = z_m$, and $\phi_7 = I$. $\bar{\phi}_k$ is the nominal value of the parameter associated with $\overline{\Delta t}_{ij}$. I is an instrumentation parameter which incorporates all of the system uncertainties such as clock accuracy.

The maximum error that vehicle dynamics and instrumentation introduce in the determination of δ_1 will be investigated in the following manner:

$$\left[\frac{\partial \Delta t_{ij}}{\partial \delta_1} \right]^2_{t=\bar{t}} (\bar{\delta}_1 - \delta_1)^2 = (\Delta t_{ij} - \overline{\Delta t}_{ij})^2 - \sum_{k=2}^{7} \left[\frac{\Delta t_{ij}}{\partial \phi_k} \right]^2_{t=\bar{t}} (\phi_k - \phi_k)^2 \ . \tag{12}$$

The various sensitivities are developed in the following subsections.

(1) Pitch Sensitivity, $\frac{\partial(\Delta t)}{\partial \delta_1}$. The pitch sensitivity can be obtained by using Equations (7) and (8).

$$\Delta t_{ij} = \frac{1}{\omega} \left\{ \arctan\left(\frac{\xi_j}{\eta_j}\right) - \arctan\left(\frac{\xi_i}{\eta_i}\right) \right\} \ . \tag{13}$$

Now, since

$$\frac{d}{dx} \tan^{-1} u = \frac{1}{1+u^2} \frac{du}{dx} \ , \tag{14}$$

the pitch sensitivity is

$$\frac{\partial(\Delta t_{ij})}{\partial \delta_1} = \frac{1}{\omega} \left\{ \frac{1}{1+\left(\frac{\xi_j}{\eta_j}\right)^2} \frac{\partial\left(\frac{\xi_j}{\eta_j}\right)}{\partial \delta_1} - \frac{1}{1+\left(\frac{\xi_i}{\eta_i}\right)^2} \frac{\partial\left(\frac{\xi_i}{\eta_i}\right)}{\partial \delta_1} \right\} \ . \tag{15}$$

Since η_i is independent of δ_1 [Equation (4)] one obtains

$$\frac{\partial(\Delta t_{ij})}{\partial\delta_1} = \frac{1}{\omega}\left\{\frac{\eta_j}{\eta_j^2 + \xi_j^2}\frac{\partial\xi_j}{\partial\delta_1} - \frac{\eta_i}{\eta_i^2 + \xi_i^2}\frac{\partial\xi_i}{\partial\delta_1}\right\} \tag{16}$$

where

$$\frac{\partial\xi_i}{\partial\delta_1} = -\sin(\delta_2)\cos(\delta_1)(x_i - x_m) - \cos(\delta_2)\cos(\delta_1)(y_i - y_m) - \sin(\delta_1)(z_i - z_m) \quad . \tag{17}$$

(2) Yaw Sensitivity, $\frac{\partial(\Delta t)}{\partial\delta_2}$. Approaching the determination of yaw sensitivity in the same manner, one obtains

$$\frac{\partial(\Delta t_{ij})}{\partial\delta_2} = \frac{1}{\omega}\left\{\frac{1}{1 + \left(\frac{\xi_j}{\eta_j}\right)^2}\frac{\partial\left(\frac{\xi_j}{\eta_j}\right)}{\partial\delta_2} - \frac{1}{1 + \left(\frac{\xi_i}{\eta_i}\right)^2}\frac{\partial\left(\frac{\xi_i}{\eta_i}\right)}{\partial\delta_2}\right\} \tag{18}$$

$$= \frac{1}{\omega}\left\{\frac{\eta_j\frac{\partial\xi_j}{\partial\delta_2} - \xi_j\frac{\partial\eta_j}{\partial\delta_2}}{\eta_j^2 + \xi_j^2} - \frac{\eta_i\frac{\partial\xi_i}{\partial\delta_2} - \xi_i\frac{\partial\eta_i}{\partial\delta_2}}{\eta_i^2 + \xi_i^2}\right\} \tag{19}$$

where

$$\frac{\partial\xi_i}{\partial\delta_2} = -\cos(\delta_2)\sin(\delta_1)(x_i - x_m) + \sin(\delta_2)\sin(\delta_1)(y_i - y_m) \tag{20}$$

$$\frac{\partial\eta_i}{\partial\delta_2} = -\sin(\delta_2)(x_i - x_m) - \cos(\delta_2)(y_i - y_m) \quad . \tag{21}$$

(3) Altitude Sensitivity, $\frac{\partial(\Delta t)}{\partial z_m}$. The altitude sensitivity is given by

$$\frac{\partial(\Delta t_{ij})}{\partial z_m} = \frac{1}{\omega}\left\{\frac{1}{1+\left(\frac{\xi_j}{\eta_j}\right)^2}\frac{\partial\left(\frac{\xi_j}{\eta_j}\right)}{\partial z_m} - \frac{1}{1+\left(\frac{\xi_i}{\eta_i}\right)^2}\frac{\partial\left(\frac{\xi_i}{\eta_i}\right)}{\partial z_m}\right\} \tag{22}$$

and since η_i is not a function of z_m

$$\frac{\partial(\Delta t_{ij})}{\partial z_m} = \frac{1}{\omega}\left\{\frac{\eta_j}{\eta_j^2+\xi_j^2}\left(\frac{\partial\xi_j}{\partial z_m}\right) - \frac{\eta_i}{\eta_i^2+\xi_i^2}\left(\frac{\partial\xi_i}{\partial z_m}\right)\right\} \tag{23}$$

where

$$\frac{\partial\xi_i}{\partial z_m} = -\cos\delta_1 \tag{24}$$

so that

$$\frac{\partial(\Delta t_{ij})}{\partial z_m} = \frac{\cos\delta_1}{\omega}\left\{\frac{\eta_i}{\eta_i^2+\xi_i^2} - \frac{\eta_j}{\eta_j^2+\xi_j^2}\right\}. \tag{25}$$

(4) Roll Rate Sensitivity, $\frac{\partial(\Delta t)}{\partial\omega}$. The roll rate sensitivity is simply

$$\frac{\partial(\Delta t_{ij})}{\partial\omega} = \frac{-1}{\omega^2}\left[\arctan\left(\frac{\xi_j}{\eta_j}\right) - \arctan\left(\frac{\xi_i}{\eta_i}\right)\right] \quad . \tag{26}$$

(5) Range Sensitivity, $\frac{\partial(\Delta t)}{\partial y_m}$. The range sensitivity is given by

$$\frac{\partial(\Delta t_{ij})}{\partial y_m} = \frac{1}{\omega}\left\{\left[\frac{\eta_j\frac{\partial\xi_j}{\partial y_m} - \xi_j\frac{\partial\eta_j}{\partial y_m}}{\eta_j^2+\xi_j^2}\right] - \left[\frac{\eta_i\frac{\partial\xi_i}{\partial y_m} - \xi_i\frac{\partial\eta_i}{\partial y_m}}{\eta_i^2+\xi_i^2}\right]\right\} \tag{27}$$

where

$$\frac{\partial \xi_i}{\partial y_m} = \cos(\delta_2)\ \sin(\delta_1) \tag{28}$$

$$\frac{\partial \eta_i}{\partial y_m} = \sin(\delta_2) \quad . \tag{29}$$

(6) Crossrange Sensitivity.

$$\frac{(\Delta t_{ij})}{\partial x_m} = \frac{1}{\omega}\left\{\left[\frac{\eta_j \frac{\partial \xi_i}{\partial x_m} - \xi_j \frac{\partial \eta_i}{\partial x_m}}{\eta_j^2 + \xi_j^2}\right] - \left[\frac{\eta_i \frac{\partial \xi_i}{\partial x_m} - \xi_i \frac{\partial \eta_i}{\partial x_m}}{\eta_i^2 + \xi_i^2}\right]\right\} \tag{30}$$

where

$$\frac{\partial \xi_i}{\partial x_m} = \sin(\delta_2)\ \sin(\delta_1) \tag{31}$$

$$\frac{\partial \eta_i}{\partial x_m} = -\cos(\delta_2) \quad . \tag{32}$$

In order to proceed further in a practical sense, some estimate of the values of $(\bar{\Phi}_k - \Phi_k)$ must be determined to provide numerical values for the terms in the summation, as well as Δt_{ij}, and $\bar{\Delta} t_{ij}$ in Equation (12). The approach taken in this investigation has been to choose a representative vehicle (ARROW), and exercise a six-degree-of-freedom computer simulation to provide some representative values.

Assuming three ground stations with coordinates (3000, 0, 0), (3000, 2000, 0), and (1000, 3000, 0), the results of the simulation of an ARROW flight indicate that the $\bar{\Phi}_k - \Phi_k$ are of the orders of magnitude indicated in Table 1. The various sensitivities are also calculated in the simulation program. Using the calculated sensitivities and the $\bar{\bar{\Phi}}_k - \Phi_k$ determined from the simulation, an estimate of the error arising from vehicle dynamics may be obtained. In the example used (ARROW), the dynamics of the vehicle introduced an error in pitch of approximately 8×10^{-4} degrees and in yaw of approximately 6×10^{-2} degrees.

TABLE 1. ERROR ANALYSIS SUMMARY

ϕ_k	Pitch System		Yaw System	
	$\lvert\partial\Delta t_{ij}/\partial\phi_k\rvert$	$\lvert\bar{\phi}_k - \phi_k\rvert$	$\lvert\partial\Delta t_{ij}/\partial\phi_k\rvert$	$\lvert\bar{\phi}_k - \phi_k\rvert$
Roll rate (°/sec)	6.702E-8	1.118	3.796E-8	4.160
x_m (ft)	9.625E-9	7.355E-5	6.639E-9	2.737E-4
y_m (ft)	2.201E-9	5.526E-2	1.462E-9	2.057E-1
z_m (ft)	8.364E-8	1.176E-3	2.370E-4	8.752E-3
Pitch (deg)	3.457E-4		1.730E-5	
Yaw (deg)	8.804E-7			

b. Conclusions

This study indicates that a three ground station system employing retroreflecting prisms (single plane corner reflectors) can provide a useful range instrumentation technique for vehicle attitude determination. Yaw sensitivity and therefore yaw accuracy will be less than pitch accuracy. Furthermore, for the particular combination of station locations investigated, the system sensitivity is a function of downrange position with an indication of severe degradation in yaw sensitivity for some combinations of flight parameters near launch. It is possible that a portable third station could be located at an optimum position for maximizing yaw sensitivity for each flight test. However, the three station system investigated combined with optical lever instrumentation for the near launch phase of flight may provide sufficiently accurate flight test data through the first 2000 feet of flight.

A number of practical problems associated with the overall system accuracy have not been addressed. Specifically, tracking accuracy, CW laser transmitter and detector characteristics, tolerance of optical components, fabrication tolerances, preflight calibration, deformation of the reflectors due to acceleration, and thermal stress were not considered. None of the above considerations appear to present insurmountable problems; however, they all will contribute to degradation of the system performance and therefore, warrant further consideration.

3. The Two Station System

The requirement of a third station can be eliminated using a two station concept which requires two single plane corner reflectors on the vehicle mounted in such a fashion that the planes of the corner reflectors (defined previously) are skewed relative to one another. As the vehicle is tracked downrange two pulses are returned to each tracking station for each revolution of the vehicle. The time intervals between the various pulses again provide the data to determine pitch, yaw, and roll attitude.

a. System Concept

The geometrical arrangement of the optical components is considered as shown in Figure 9. The two single plane corner reflectors are located at the same axial position on the vehicle but are separated by a polar angle equal to 2β. Further, they are inclined relative to the roll axis by angles γ_1 and γ_2. As a result the planes of the two corner reflectors are skewed with respect to both the roll axis and each other as shown in Figure 10. When the vehicle rotates to a position so that a retroreflection plane passes through a transmitter/detector ground station, a pulse is retroreflected to the ground station. Consider a vehicle fixed in space and rotating at a frequency ω. Referring to Figure 9, it may be seen that the time interval between retroreflected pulses are given by

$$2\pi\omega\Delta t_{12} = 2\beta + \theta_1 + \theta_2 \quad . \tag{33}$$

Note from Figure 10 that

$$\tan \sigma = \frac{R}{B} \quad ,$$

where σ is called the laser aspect angle and

$$\tan \gamma_i = \frac{C_i}{B}$$

so that

$$\theta_i = \arcsin \frac{(C_i)}{(R)} = \arcsin \frac{(\tan \gamma_i)}{(\tan \sigma)} \quad .$$

Equation (33) may be rewritten as

$$2\pi\omega\Delta t_{12} - 2\beta = \arcsin \left| \frac{\tan \gamma_1}{\tan \sigma} \right| + \arcsin \left| \frac{\tan \gamma_2}{\tan \sigma} \right| \quad . \tag{34}$$

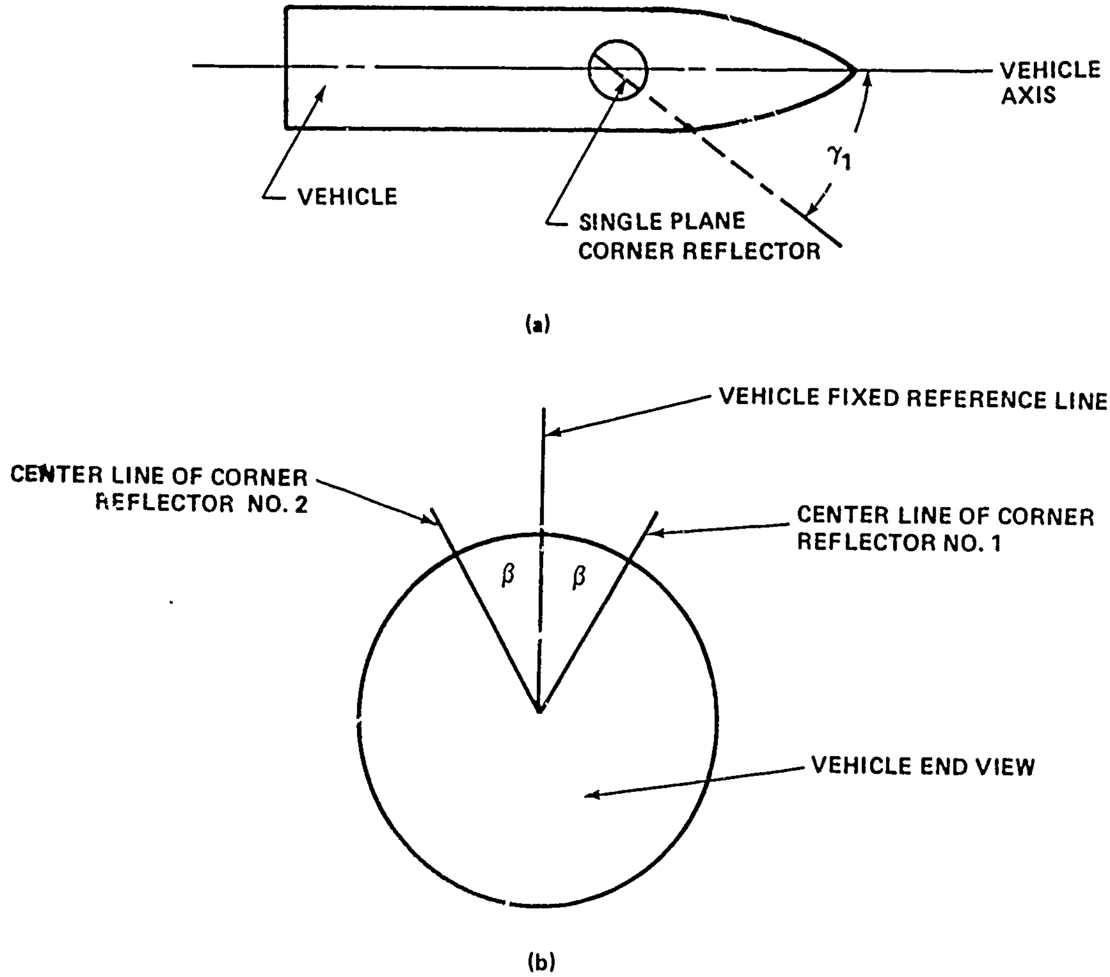

Figure 9. Corner reflector geometry relative to the vehicle.

Taking the sin of both sides, one obtains

$$\sin \left\{2(\pi\omega\Delta t_{12} - \beta)\right\} = \frac{\tan \gamma_1}{\tan \sigma} \cos \left\{\arcsin \left(\frac{\tan \gamma_2}{\tan \sigma}\right)\right\} + \frac{\tan \gamma_2}{\tan \sigma} \cos \left\{\arcsin \left(\frac{\tan \gamma_1}{\tan \sigma}\right)\right\} . \tag{35}$$

Noting the geometry shown in Figure 11, the following expression is obtained:

$$\cos \left\{\arcsin \left(\frac{\tan \gamma_i}{\tan \sigma}\right)\right\} = \frac{(\tan^2 \sigma - \tan^2 \gamma_i)^{1/2}}{\tan \sigma} , \tag{36}$$

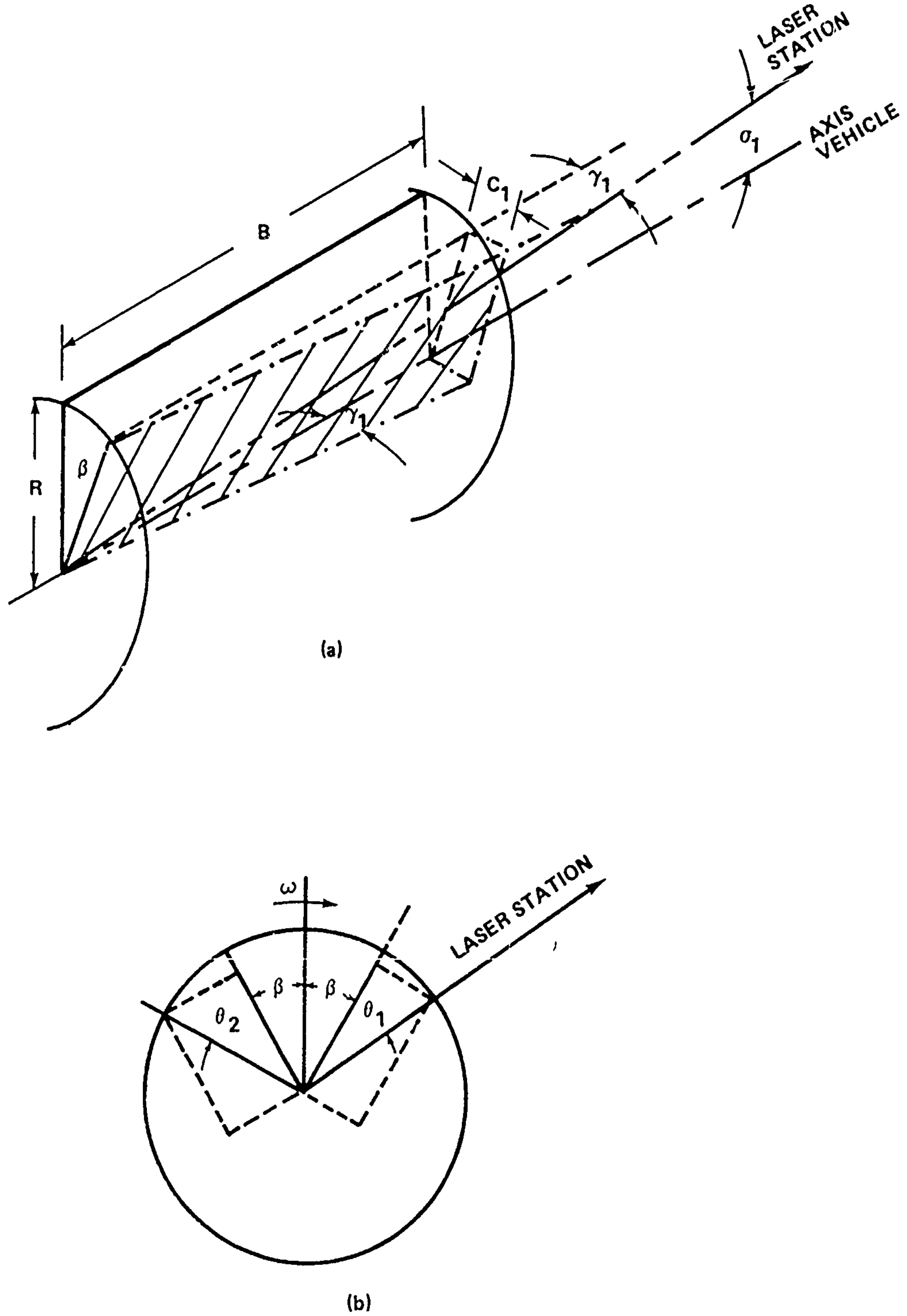

Figure 10. Geometry to determine laser aspect angle.

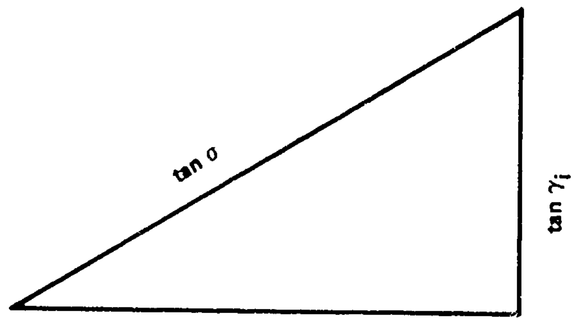

Figure 11. Geometry for substituion.

so that (35) may be expressed as

$$\sin\{2(\pi\omega\Delta t_{12} - \beta)\} = \frac{\tan\gamma_1}{\tan\sigma}\,\frac{(\tan^2\sigma - \tan^2\gamma_2)^{1/2}}{\tan\sigma} + \frac{\tan\gamma_2}{\tan\sigma}\,\frac{(\tan^2\sigma - \tan^2\gamma_1)^{1/2}}{\tan\sigma} , \tag{37}$$

or

$$\tan^2\sigma \sin\{2(\pi\omega\Delta t_{12} - \beta)\} = \tan\gamma_1\,(\tan^2\sigma - \tan^2\gamma_2)^{1/2} + \tan\gamma_2\,(\tan^2\sigma - \tan^2\gamma_1)^{1/2} . \tag{38}$$

Now taking the cos of both sides of (34) one has

$$\cos\{2(\pi\omega\Delta t_{12} - \beta)\} = \frac{(\tan^2\sigma - \tan^2\gamma_1)^{1/2}\,(\tan^2\sigma - \tan^2\gamma_2)^{1/2} - \tan\gamma_1\tan\gamma_2}{\tan^2\sigma} \tag{39}$$

This may be solved for $(\tan^2\sigma - \tan^2\gamma_1)^{1/2}$ to yield

$$(\tan^2\sigma - \tan^2\gamma_1)^{1/2} = \cos\frac{\{2(\pi\omega\Delta t_{12} - \beta)\}\tan^2\sigma + \tan\gamma_1\tan\gamma_2}{(\tan^2\sigma - \tan^2\gamma_2)^{1/2}} . \tag{40}$$

When substituted into Equation (38) one obtains

$$\sin\{2(\pi\omega\Delta t_{12} - \beta)\} \tan^2\sigma = \tan\gamma_1 (\tan^2\sigma - \tan^2\gamma_2)^{1/2}$$

$$+ \tan\gamma_2 \left\{\frac{\cos\{2(\pi\omega\Delta t_{12} - \beta)\} \tan^2\sigma + \tan\gamma_1 \tan\gamma_2}{(\tan^2\sigma - \tan^2\gamma_2)^{1/2}}\right\} . \quad (41)$$

Multiply both sides by $(\tan^2\sigma - \tan^2\gamma_2)^{1/2}$ gives

$$(\tan^2\sigma - \tan^2\gamma_2)^{1/2} \sin\{2(\pi\omega\Delta t - \beta)\} \tan^2\sigma = \tan\gamma_1 \tan^2\sigma$$

$$+ \tan\gamma_2 \cos\{2(\pi\omega\Delta t_{12} - \beta)\} \tan^2\sigma \quad . \quad (42)$$

Dividing through by $\sin(2(\pi\omega\Delta t_{12} - \beta)) \tan^2\sigma$ and squaring both sides yield

$$\tan^2\sigma - \tan^2\gamma_2 = \frac{\{\tan\gamma_1 + \tan\gamma_2 \cos\{2(\pi\omega\Delta t_{12} - \beta)\}\}^2}{\sin^2\{2(\pi\omega\Delta t_{12} - \beta)\}} . \quad (43)$$

This may be solved for $\tan^2\sigma$:

$$\tan^2\sigma = \Big\{\tan^2\gamma_1 + 2\tan\gamma_1 \cos\{2(\pi\omega\Delta t_{12} - \beta)\} \tan\gamma_2$$

$$+ \tan^2\gamma_2 \cos^2\{2(\pi\omega\Delta t_{12} - \beta)\} + \tan^2\gamma_2 \sin^2\{2(\pi\omega\Delta t_{12} - \beta)\}\Big\}$$

$$* \frac{1}{\sin^2(2(\pi\omega\Delta t_{12} - \beta))} . \quad (44)$$

Alternately

$$\tan^2\sigma = \left\{\tan^2\gamma_1 + 2\tan\gamma_1 \tan\gamma_2 \cos\{2(\pi\omega\Delta t_{12} - \beta)\} + \tan^2\gamma_2\right\} \quad (45)$$

$$* \frac{1}{\sin^2\{2(\pi\omega\Delta t_{12} - \beta)\}} .$$

Take the square root of both sides, let $\frac{\tan \gamma_2}{\tan \gamma_1} = A$, and $2\pi\omega\Delta t\ 2\beta = \alpha$

yielding $\tan \sigma = \tan \gamma_1 \ (1 + A^2 + 2A \cos \alpha)^{1/2} / \sin \alpha.$ (46)

It is apparent that Equation (46) provides a relationship for the laser aspect angle (σ) which is the angle between the roll axis of the vehicle and the vector joining the tracking station and the vehicle. This relationship is in terms of fixed geometric parameters ($\gamma_1, \gamma_2, \beta$) and two parameters which are measured, i.e., ω (determined from the time interval between pulses returned to one ground station from one corner reflector) and Δt (determined from the time interval between pulses from the two different corner reflectors).

b. Data Reduction

For simplicity, a configuration will be considered which has one reflector parallel to the roll axis, a second reflector skewed an angle γ to the roll axis, and two laser tracking stations. Two types of return pulses are received at the stations. The time interval for return pulses from the unskewed single plane corner reflectors to the two ground stations is given by Equation (9) which is rewritten as

$$\Delta t_{12} = \frac{1}{\omega} \left\{ \arctan \left[\frac{-\sin(\delta_2)\sin(\delta_1)(x_2 - x_m) - \cos(\delta_2)\sin(\delta_1)\, y_2 - y_m) + \cos(\delta_1)(z_2 - z_m)}{\cos(\delta_2)(x_2 - x_m) - \sin(\delta_2)(y_2 - y_m)} \right] \right.$$
$$\left. - \arctan \left[\frac{-\sin(\delta_2)\sin(\delta_1)(x_1 - x_m) - \cos(\delta_2)\sin(\delta_1)(y_1 - y_m) + \cos(\delta_1)(z_1 - z_m)}{\cos(\delta_2)(x_1 - x_m) - \sin(\delta_2)(y_1 - y_m)} \right] \right\} \quad (47)$$

Referring to Figure 5 it may be seen that the aspect angles can be expressed in terms of $\eta\omega\xi$ coordinates such as

$$\tan \sigma_1 = \frac{\omega_1}{\sqrt{\eta_1^2 + \xi_1^2}} \quad . \quad (48)$$

Substituting in Equation (15) and using the expressions for η_i, ω_i, ξ_i given previously (Equations (4), (5), and (6)), one obtains

$$\tan \frac{\sigma_1 (1 + A^2 + 2A \cos)^{1/2}}{\sin \alpha} = \frac{\sin\delta_2 \cos\delta_1 (x_1 - x_m) + \cos\delta_2 \cos\delta_1 (y_1 - y_m) + \sin\delta_1 (z_1 - z_m)}{[\{\cos\delta_2 (x_1 - x_m) - \sin_2 (y_1 - y_m)\}^2 + \{-\sin\delta_2 \sin\delta_1 (x_1 - x_m) - \cos\delta_2 \sin\delta_1 (y_1 - y_m) + \cos\delta_1 (z - z_m)\}^2]^{1/2}} \cdot (49)$$

It is apparent that Equations (47) and (49) form a pair of simultaneous equations in two unknowns, i.e., δ_1 and δ_2. As in the three station system a scheme such as the Newton-Raphson technique is required to determine δ_1 and δ_2. Recent studies indicate that multivalued solutions exist for this pair of equations. Although it appears that physical constraints may be used to eliminate at least one of the possibilities, a definitive statement in this regard cannot be made at this time. It should be noted that it may be possible to follow a unique solution curve if continuous data from launch is available. This area is currently under investigation.

c. Sensitivity and Accuracy

A detailed investigation of sensitivity and accuracy particularly with reference to vehicle dynamics has not been completed for the two station concept. A study of the errors associated with a determination of σ using Equation (46) is presented. Rewriting Equation (33), one obtains

$$\Delta t_{12} = \{2\beta + \theta_1 + \theta_2\} \frac{1}{2\pi\omega} \tag{50}$$

or alternately

$$\Delta t_{12} = \left\{2\beta + \arcsin\left(\frac{\tan\gamma_1}{\tan\sigma}\right) + \arcsin\left(\frac{\tan\gamma_2}{\tan\sigma}\right)\right\} \frac{1}{2\pi\omega} \cdot \tag{51}$$

The sensitivity of the time interval to changes in laser aspect angle (σ) is

$$\frac{\partial(\Delta t)}{\partial\sigma} .$$

Now

$$\frac{d}{dx}(\arcsin u) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}, \left(-\frac{\pi}{2} \le \arcsin u \le \frac{\pi}{2}\right) ,$$

so that

$$\frac{\partial(\Delta t)}{\partial\sigma} = \frac{1}{2\pi\omega}\left\{\frac{1}{\sqrt{1-\left(\frac{\tan\gamma_1}{\tan\sigma}\right)^2}}\frac{d\left(\frac{\tan\gamma_1}{\tan\sigma}\right)}{d\sigma} + \frac{1}{\sqrt{1-\left(\frac{\tan\gamma_2}{\tan\sigma}\right)^2}}\frac{d\left(\frac{\tan\gamma_2}{\tan\sigma}\right)}{d\sigma}\right\}. \tag{52}$$

Now

$$\frac{d\left(\frac{1}{\tan\sigma}\right)}{d\sigma} = -\csc^2 \quad .$$

Therefore

$$\frac{\partial(\Delta t)}{\partial\sigma} = \left\{\frac{-\csc^2\sigma\ \tan\sigma}{2\pi\omega}\ \frac{\tan\gamma_1}{(\tan^2\sigma - \tan^2\gamma_1)^{1/2}} + \frac{\tan\gamma_2}{(\tan^2\sigma - \tan^2\gamma_2)^{1/2}}\right\}. \qquad (53)$$

A number of potential error sources are involved in a determination of σ using Equation (46). The magnitude of the combined errors may be investigated using the equation

$$(\sigma_m - \bar{\sigma})^2 = \left(\frac{\partial\sigma}{\partial\gamma_1}\delta\gamma_1\right)^2 + \left(\frac{\partial\sigma}{\partial\gamma_2}\delta\gamma_2\right)^2 + \left(\frac{\partial\sigma}{\partial\omega}\delta\omega\right)^2 + \left(\frac{\partial\sigma}{\partial\beta}\delta\beta\right)^2 + \left(\frac{\partial\sigma}{\partial\Delta t}\delta(\Delta t)\right)^2 \qquad (54)$$

where σ_m is the measured value and $\bar{\sigma}$ is the actual value. Expressions for the various sensitivities are developed. From Equation (46) one obtains

$$\sigma = \arctan\left\{\tan\gamma_1(1 + A^2 + 2A\cos\alpha)^{1/2}/\sin\alpha\right\} \quad . \qquad (55)$$

The sensitivity of σ to γ_1 is

$$\frac{\partial\sigma}{\partial\gamma_1} = \frac{\sin\alpha\ \sec^2\gamma_1}{\sin^2\alpha + \tan^2\gamma_1(\eta)}\left\{\eta^{1/2} \cdot A\eta^{-1/2}\left\{\cos\alpha + A\right\}\right\} \qquad (56)$$

where $\eta = (1 + A^2 + 2A\cos\alpha)$. The sensitivity of the laser aspect angle to the inclination of the second corner reflector is

$$\frac{\partial\sigma}{\partial\gamma_2} = \frac{\sin\alpha\ \sec^2\gamma_2(\eta^{-1/2})}{\sin^2\alpha + \eta\tan^2\gamma_1}\left\{A + \cos\alpha\right\} \quad . \qquad (57)$$

The sensitivity of the laser aspect angle to roll rate is

$$\frac{\partial\sigma}{\partial\omega} = \frac{-2\pi\Delta t \tan\gamma_1}{\sin^2\alpha + \eta\tan^2\gamma_1}\left\{\frac{A\sin^2\alpha}{\eta^{1/2}} + \eta^{1/2}\cos\alpha\right\} \quad . \tag{58}$$

Sensitivity of the laser aspect angle to time interval variation is given by

$$\frac{\partial\sigma}{\partial(\Delta t)} = \frac{-2\pi\omega\tan\gamma_1}{\sin^2\alpha + \eta\tan^2\gamma_1}\left\{\frac{A\sin^2\alpha}{\eta^{1/2}} + \eta^{1/2}\cos\alpha\right\} \quad . \tag{59}$$

The sensitivity of laser aspect angle to angular spacing between the retroreflectors (β) is

$$\frac{\partial\sigma}{\partial\beta} = \frac{2\tan\gamma_1}{\sin^2\alpha + \eta\tan^2\gamma_1}\left\{\frac{A\sin^2\alpha}{\eta^{1/2}} + \eta^{1/2}\cos\alpha\right\} \quad . \tag{60}$$

Combining Equations (54) and (56) through (60), one obtains

$$\begin{aligned}(\sigma_m - \bar{\sigma})^2 = {} & \left(\frac{\sin\alpha\sec^2\gamma_1}{\sin^2\alpha + \eta\tan^2\gamma_1}\right)^2\left(\eta^{1/2} - \frac{A}{\eta^{1/2}}\{\cos\alpha + A\}\right)^2(\delta\gamma_1)^2 \\ & + \left(\frac{\sin\alpha\sec^2\gamma_2\eta^{-1/2}}{\sin^2\alpha + \eta\tan^2\gamma_1}\right)^2(A + \cos\alpha)^2(\delta\gamma_2)^2 \\ & + \left(\frac{2\pi\Delta t\tan\gamma_1}{\sin^2\alpha + \eta\tan^2\gamma_1}\right)^2\left(\frac{A\sin^2\alpha}{\eta^{1/2}} + \eta^{1/2}\cos\alpha\right)^2(\delta\omega)^2 \\ & + \left(\frac{2\pi\omega\tan\gamma_1}{\sin^2\alpha + \eta\tan^2\gamma_1}\right)^2\left(\frac{A\sin^2\alpha}{\eta^{1/2}} + \eta^{1/2}\cos\alpha\right)^2(\delta\Delta t)^2 \\ & + \left(\frac{2\tan\gamma_1}{\sin^2\alpha + \eta\tan^2\gamma_1}\right)^2\left(\frac{A\sin^2\alpha}{\eta^{1/2}} + \eta^{1/2}\cos\alpha\right)^2(\delta\beta)^2 \quad .\end{aligned} \tag{61}$$

d. Conclusions

Although a complete system study for the two station concept has not been completed the studies conducted to date indicate that this approach may be a cost effective range instrumentation technique. Additional studies, similar to those described for the three station system, are warranted. It should be noted that if the technique actually works as described here all of the advantages originally envisioned with the use of the three station system would accrue to the two station system.

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